



Frequency-magnitude Distribution and Fractal Dimension of the Seismicity along the Sumatra-Andaman Subduction Zone: A Temporal Comparison

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ABSTRACT

The Sumatra-Andaman subduction zone (SASZ) is formed by the Indo-Australian and Eurasian plates collision. Among the tectonic subduction zone in the world, the SASZ is one of the active subduction zones which has generated continuously many great earthquakes. The main aim of this research is to investigate temporally the seismicity by two statistical-seismological methods, i.e., fractal dimension (D_c value) and frequency-magnitude distribution (b value). In addition, this research focused on two groups of seismicity data which are i) before and ii) after earthquake declustering process. In individual dataset, both D_c and b values were investigated temporally at the epicenters of 6 earthquakes with magnitude ≥ 8.0 . The results according to both after-declustering and before-declustering dataset revealed that b value was low before the earthquake occurred in all 6 earthquake case studies approximately 0.91-0.36 and 0.79-0.56. For D_c value of both after and before declustering datasets are nearly reached 2.0. It means that the pattern of earthquake occurrences distribute in the areal or plane source. In addition according to temporal investigation, all the B- D_c relationship of 6 case studies along the SASZ is defined as negative relationship. Finally, the results analyzed from after- and before-declustering dataset were similar which meaningful for B- D_c investigation.

Keywords: Earthquake Catalogue; Earthquake Declustering; b value; Fractal Dimension; Sumatra-Andaman Subduction Zone.

1. Introduction

The Sumatra-Andaman subduction zone (SASZ) is formed by the Indo-Australian and Eurasian plate collision, which continually subducts 50 mm/yr at strike $\sim N30E$ beneath the Burma microplate of Eurasia. (Roy et al., 2011). The SASZ is one of the most hazardous and active subduction zones that has generated many great earthquakes such as M_w 7.1 in 1984, M_w 9.2 in 2004, M_w 7.2, 8.6 in 2005, M_w 7.3 in 2008, M_w 7.5 in 2009, M_w 7.8, 7.5 in 2010 and M_w 8.6, 8.2 in 2012 (Figure 1). In 2004, SASZ

generated the M_w 9.2 on December 26th 2004. Both earthquake and tsunami hazards affected several countries where settle surrounding the Indian Ocean, especially southern part of Thailand (Waldhauser et al., 2012). The tsunami made many people lost their lives, their beloved family's members and their assets. For example, tsunami in Thailand, there were 5,395 people died, more than 2,817 people missed and 7,000 people displaced. Moreover, 128,645 Indonesians, 35,322 Sri Lankans and 12,405 Indians also died because of tsunami (USAID, 2005). When



the earthquake and tsunami generate, it is impossible to stop them. However, finding out the solution to help people to be safe from earthquake and tsunami is possible.

Death and Losing always happens after the earthquake and tsunami. This problem inspired the scientist to think of many ways to solve this problem. One interesting way is to forecast when the earthquake is going to generate. Nowadays, the study of forecasting has many methods such as b value (Nuannin et al., 2005), z value (Wiemer and Wyss., 1994), RTL (Huang et al., 2001), pattern informations (Holliday et al., 2006a), fractal dimension (Bhattacharya and Kayal., 2003) etc. Through this research, we chose to focus on 2 statistical methods; frequency-magnitude distribution (b value) and fractal dimension (Dc value). The reasons of choosing these methods are i) b value is the popular method and it relates to tectonic stress accumulated in any specific region (Mogi, 1967; Scholz, 1968), ii) Dc value indicates the pattern of seismic occurrence, i.e., point, line, area, or volume sources (Bayrak and Bayrak., 2012).

The research is going to focus on comparing the earthquake that generate from the change of tectonic by using b value and Dc value. In addition, Gutenberg and Richter (1944) presented that b value used to study about the stress of area from the relation of the frequency-magnitude distribution. For fractal dimension is the study of fragmentation of the earthquake by consider from the distance between the epicenters of earthquakes (Bayrak and Bayrak., 2012).

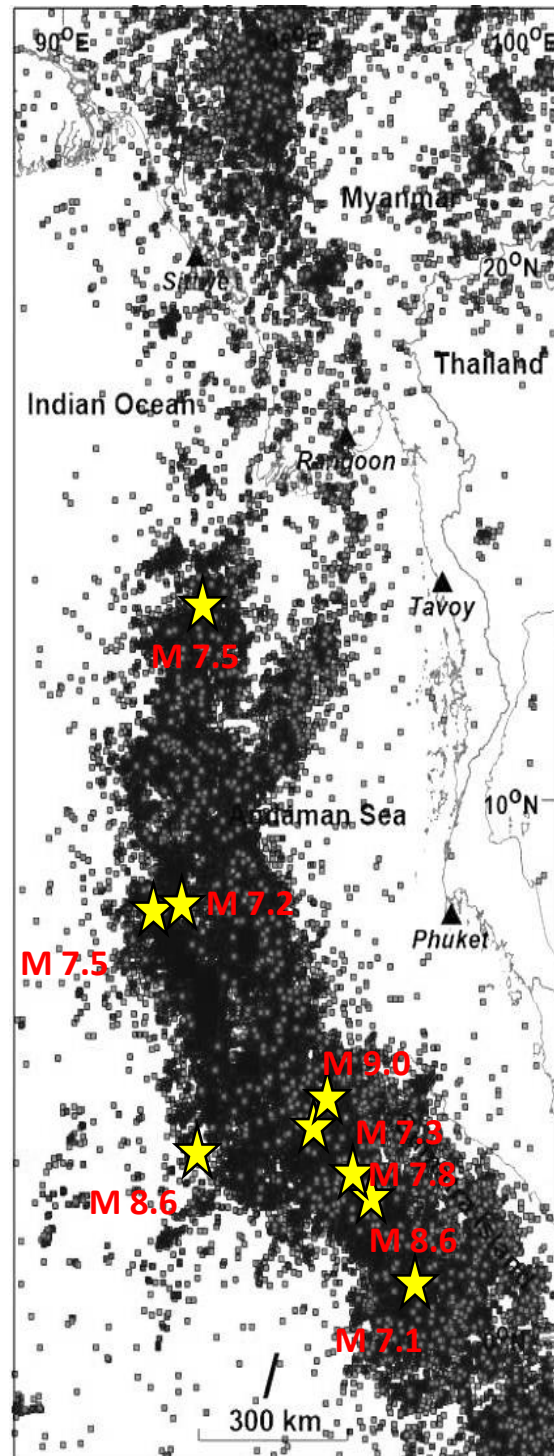


Figure 1. Map of SASZ showing epicentral distributions of earthquakes before (black circles) and after declustering (grey circles) according to Gardner and Knopoff (1974)'s algorithm.

2. Data set

2.1. Earthquake catalogue collection

The earthquake's data was collected from 2 sources: i) The International Seismological Center (ISC) and ii) the US National Earthquake Information Center (NEIC). The sum of recorded earthquake data from both sources was approximately 66,536 events recorded during 1965-2016 (Figure 1). In detail, the recorded date illustrated that there were 1.0-9.2 earthquake magnitude scales and they were reported in the data of the body-wave magnitude scale (mb), the surface wave magnitude (M_s), and the local magnitude (MI). These data would be homogenized the earthquake magnitude scales with earthquake events recorded in moment magnitude (M_w).

2.2. Earthquake Declustering

The main shock directly refers to the released tectonic stress which uses to analyze process. So, analysis of seismic data will remove foreshock and aftershock

because they are meaningless in the seismotectonic investigations.

The declustering process, was applied in the ZMAP program (Wiemer, 2001) that the released tectonic stress could remove foreshock and aftershock from 66,536 to 3,632 earthquake events (Figure 1).

In order to check the possibility of using dataset, this research study design to use 2 groups of dataset in analysis which are i) before-declustering and ii) after-declustering dataset. In individual dataset, the case study will be in 6 earthquake events with $M_w \geq 8.0$ and the earthquake data will be collected from 300-km radius extended from the epicenters of each earthquake case study. After that, both 2 dataset were analyzed temporally the frequency-magnitude distribution and fractal dimension as demonstrated in Figure 2 and summarized in Table 1.

Table 1. List of 6 earthquake case studies and some results of b and Dc values evaluated from the bulk seismicity data withn 300-km radius from each epicenter.

No.	long	lat	d/m/y	depth	M_w	After declustering			Before declustering		
						EQ	b	Dc	EQ	b	Dc
1	94.26	3.09	26/12/2004	29	9	10943	0.722	2.13	204	0.542	2.24
2	95.98	3.29	26/12/2004	30	8.9	14525	0.732	2.15	318	0.553	2.24
3	97.07	1.67	28/3/2005	26	8.6	13564	0.749	2.23	414	0.591	2.24
4	100.99	-3.78	12/9/2007	24	8.5	9199	0.71	2.05	369	0.881	2.35
5	92.82	2.35	11/4/2012	46	8.6	2998	0.788	2.04	71	0.454	2.1
6	92.31	0.9	11/4/2012	55	8.2	767	0.631	1.69	44	0.375	1.91

3. b value and Fractal Dimension

3.1. b value

The equation of the frequency-magnitude distribution (FMD) (Gutenberg and Richter, 1944) was equation (1) :

$$\text{Log}N = a - bM \quad (1)$$

where N is the cumulative number of earthquake, value of a is the y-intercept, b is the slope and M is a magnitude of earthquake recognized.

Tectonically, b value indicates to tectonic stress (Mogi 1967; Scholz 1968). The low b values shows that there is a high stress area and it means that the area is risk to have earthquake in the future.

For example, the analysis of spatial data in SASZ, they were recorded during 1964-2016. The data set have earthquake distribution of 3,632 earthquakes. The total of b value has the magnitude of completeness (Mc) at 4.7 and the FMD plots for this study are $a = 4.66$ and $b = 0.668 \pm 0.01$ (Figure 2a).

3.2. Dc value

The correlation dimension was defined by Grassberger and Procaccia (1983) that it used to measure the space of a set of points in the case of the earthquake is epicenters. Dc value indicate seismic pattern of area and it can estimate by using the correlation dimension as the following equation (2) to (4) (Roy et al., 2011) :

$$C_r = \frac{2}{N(N-1)} N_{(R<r)} \quad (2)$$

where N is the number of earthquake that use for analysing, $N_{(R<r)}$ is the number of earthquakes at distance $R < r$ and r is the distance between two events of earthquake. The following shows the relation of the fractal dimension (Bayrak and Bayrak., 2012) :

$$C_r \approx r^{D_c} \quad (3)$$

where C_r is the correlation function, D_c is a fractal dimension. Practically, we can get the fractal dimension from the slope of the graph. For the distance (r) between two events (θ_1, ϕ_1) and (θ_2, ϕ_2) is calculated by using a spherical triangle as given by Hirata (1989) :

$$r = \cos^{-1}[\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos(\phi_1 - \phi_2)] \quad (4)$$

where θ is the latitudes of event 1 and event 2, ϕ is the longitudes of event 1 and event 2.

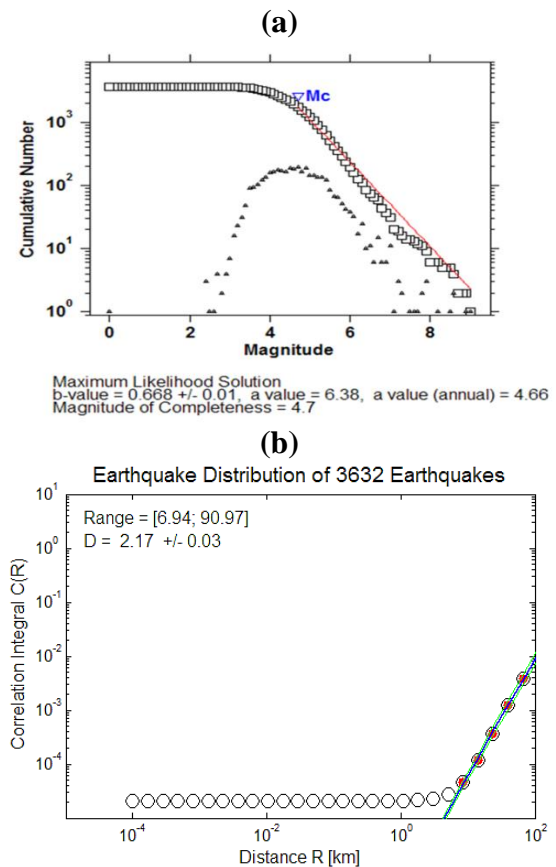


Figure 2. (a) FMD plot of the seismicity data recorded Mw 4.0 during 1964-2016. Triangles indicate the number of earthquakes of each magnitude; squares represent the cumulative number of earthquakes equal to or larger than each magnitude. The solid line is the line of best fit according to Woessner and Wiemer (2005). Mc is defined as the magnitude of completeness. (b) Graphs showing relationship between $\log(C(r))$ and $\log(r)$ of after declustering earthquake data. The slopes of linear fit (solid black lines) are the fractal dimension (DC).

To divide is the characteristics of a fault system by Aki (1981). A value of Dc close to 3 signifies that a volume of the crust, a value close to 2 signifies that a plane and a value close to 1 mean line sources (Roy et al., 2011).

Regarding to fractal dimension investigation, the overall area (SASZ)

showed that the D_c value was 2.17 ± 0.03 and distance range was 6.94 kilometer to 90.97 kilometer (Figure 2b). In the part of temporal analysis was presented in Figure 3. The graph showed the overall study in b value which were 0.9 (Figure 3a).

From Figure 3b, the D_c value was close to 2 which was indicated that characteristic of earthquake in SASZ was plane seismogenic source. The relationship of b value and D_c value was negative, $D_c = -0.20b + 2.22$ and correlation coefficient (r^2) was 0.058 (Figure 3c).

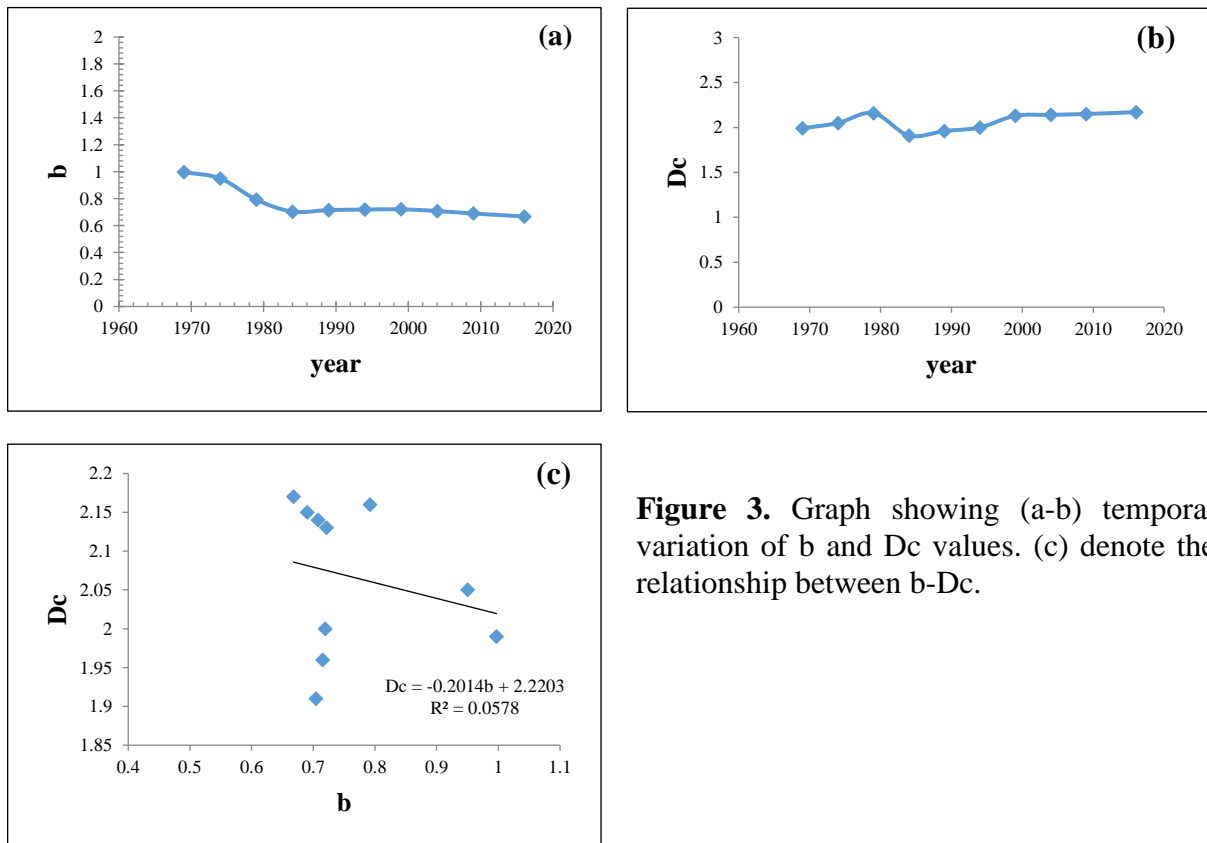


Figure 3. Graph showing (a-b) temporal variation of b and D_c values. (c) denote the relationship between b- D_c .

4. Temporal Investigation of Specific Earthquake Event

From overall study area, it can be divided into 6 earthquakes. Each event has the earthquake up to M_w 8.0 and 2 groups of data which are, i) after declustering and ii) before declustering. This research is analyzed by using b value and D_c value.

4.1. The result of after-declustering dataset

In the event 1 of Table 1, the earthquake was generated in the north of Sumatra-Andaman island and occurred in 2004, M_w 9.0. At the epicenter of the M_w -9.0 earthquake, it revealed low b value at

0.67 and it was decrease in 2003. In 2004, the b value decrease down to 0.59. The fractal dimension, D_c value, was fluctuated at range of 1.85-2.21 while in 1998, it was remained steady at 2.21 (Figure 4a). Next, event 2 was aftershock of the event 1. It generated in the north of Sumatra-Andaman Island and occurred in 2004, M_w 8.9. In 1988, the b value was 0.93 and it was continuously decrease to 0.61 in 2004. The fractal dimension, D_c value, was increase from 1988 to 1993. It started remained steady of D_c value in 1993-2004 at the range of 1.38-2.16 (Figure 4b).

Third, event 3 generated in the north of SASZ. It occurred in 2005, M_w 8.6. In 1988, the b value was 0.70 in 1988 and it was slightly decrease to 0.65 in 2005. The fractal dimension, D_c value, was continually increase from 1988 to 2005 at range of D_c value was 1.75-2.26 (Figure 4c). Forth, event 4 generated in the south of Sumatra-Andaman and occurred in 2007, M_w 8.5. The b value was fluctuated and range of b value was 1.28-1.16. From 1990 to 2007, the b value was continuously decrease to 0.91. The fractal dimension, from 1973 to 2007, the D_c value was continuously increase at range of 1.75-2.95 (Figure 4d).

Fifth, event 5 generated in the Indian Ocean and occurred in 2012, M_w 8.6. From 2000 to 2010, the b value was continuously decrease to 0.79-0.54. Also, in 2012, the b value is dramatically decrease to 0.45. The fractal Dimension, from 2000-2012, the D_c value was slightly increase at range of 1.98-2.24 (Figure 4e).

And the last, event 6 generated earthquake as same as event 5 and occurred in 2012, M_w 8.2. In 2000 to 2010, the b value was remained steady in the range 0.49-0.47. Before 2012, the b value was dramatically decrease to 0.36. The fractal dimension, D_c value, was decrease from 2000 to 2010 at range of 1.69-1.96. In 2012, the D_c value was increase at 1.92 (Figure 4f).

4.2. The result of before-declustering dataset

First, event 1, the b value was continuously decrease to range of 0.94-0.83 from 1978 to 1993. The b value was more remained steady at range of 0.83-0.76 in 1993 to 2004. The fractal dimension, D_c value, was decrease from 1978 to 1983 and range of D_c value was range of 1.99-2.16. The D_c value was

continuously increase to range of 1.99-2.24 from 1983 to 2003. In 2004, the D_c value was continually decrease to 2.17 (Figure 4a). Next, event 2, the b value was fluctuated at the range of 1.20-0.84 from 1968 to 1988. After that, it was more remained steady at range of 0.87-0.75 in 1988 to 2004. The fractal dimension, D_c value, was fluctuated at range of 1.76-2.16 from 1968 to 1983. The D_c value was continuously increase to range of 0.75-0.87 from 1983 to 2004 (Figure 4b).

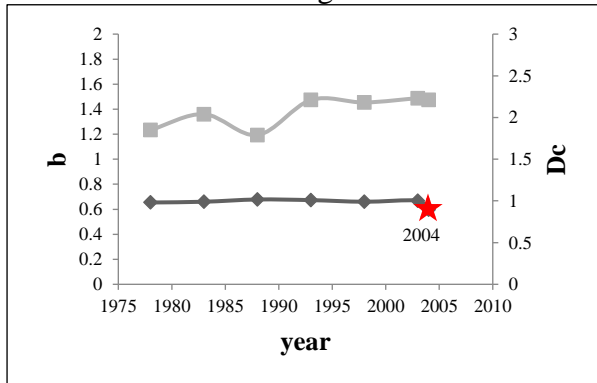
Next, event 3, the b value was fluctuated at range of 1.35-0.77 from 1968 to 1993. from 1993 to 2005, the b value was more remained steady at range of 0.86-0.77. The fractal dimension, D_c value was fluctuated at range of 1.63-2.18 from 1968 to 1983 and it was continuously increase at range of 2.08-1.73 from 1983 to 2005 (Figure 4c). Forth, event 4, the b value was continuously decrease to range of 1.34-0.95 from 1973 to 1998. From 1998 to 2000, the b value was dramatically decrease to 0.74 and in 2007, the b value was 0.75. The fractal dimension, D_c value was remained steady at range of 1.91-2.16 from 1973 to 2007 (Figure 4d).

Fifth, event 5, the b value was fluctuated from 1995 to 2005 at range of 0.89-0.62. From 2005 to 2012, the b value was continuously decrease to 0.79. The fractal dimension, D_c value, was remained steady at 1.99 from 1995 to 2005. The D_c value was decrease down to the range of 1.58-1.99 from 2005 to 2012 (Figure 4e).

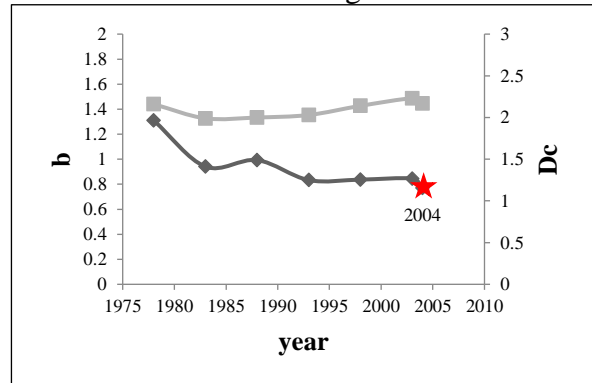
And the last event, the b value was more remained steady and range of b value was 0.67-0.56. The fractal dimension, D_c value, was more remained steady and range of it was 1.59-1.69 from 2005 to 2012 (Figure 4f).

(a) Event 1

Dataset after declustering

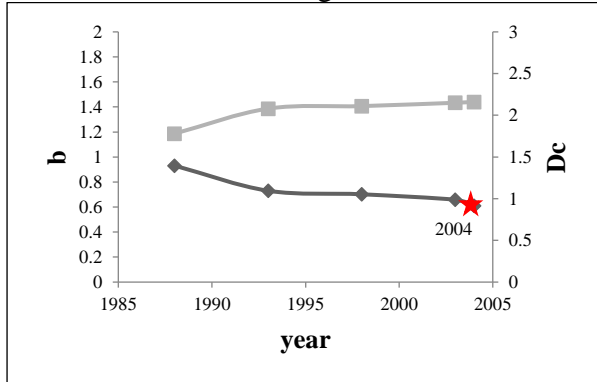


Dataset before declustering

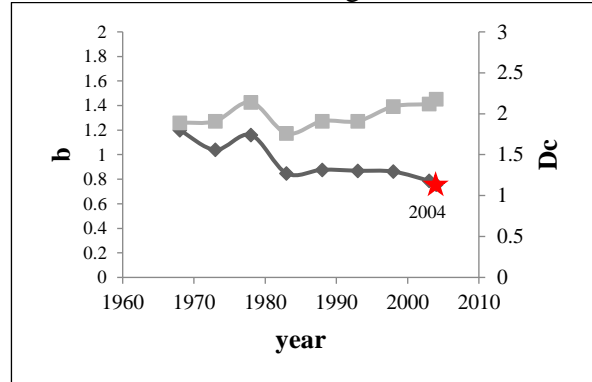


(b) Event 2

Dataset after declustering

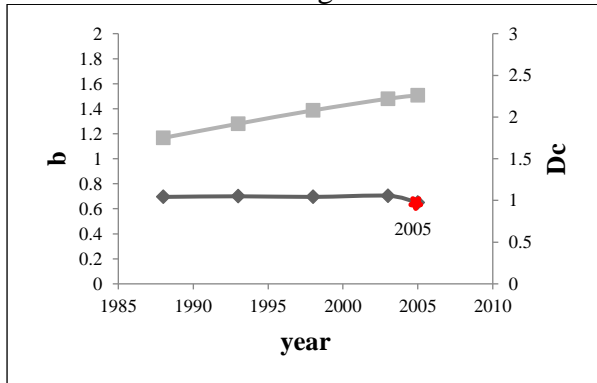


Dataset before declustering

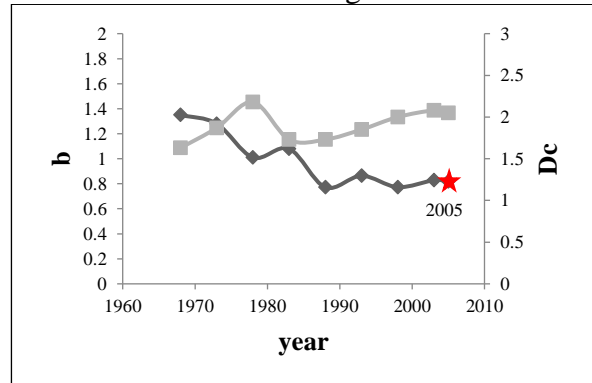


(c) Event 3

Dataset after declustering



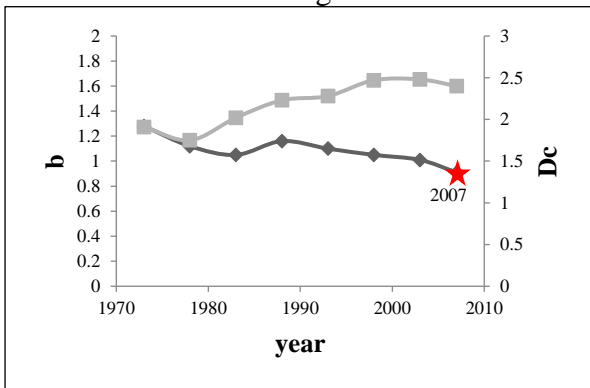
Dataset before declustering



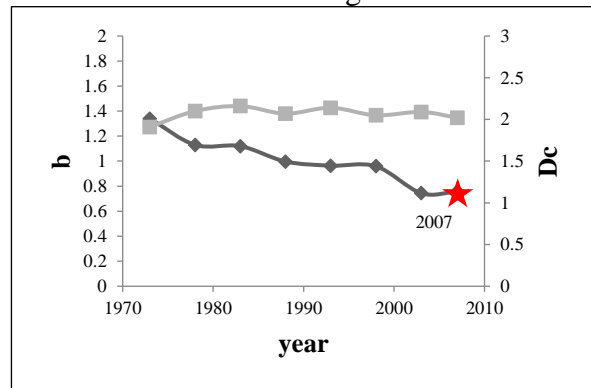


(d) Event 4

Dataset after declustering

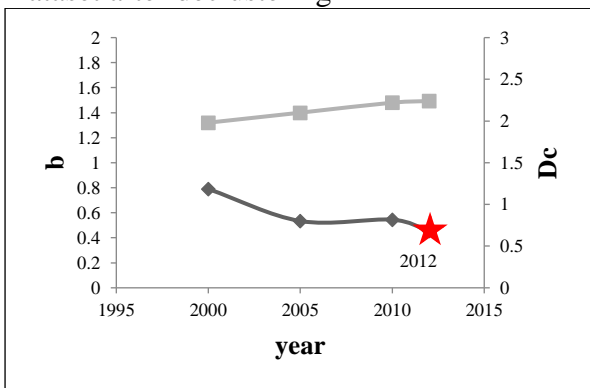


Dataset before declustering

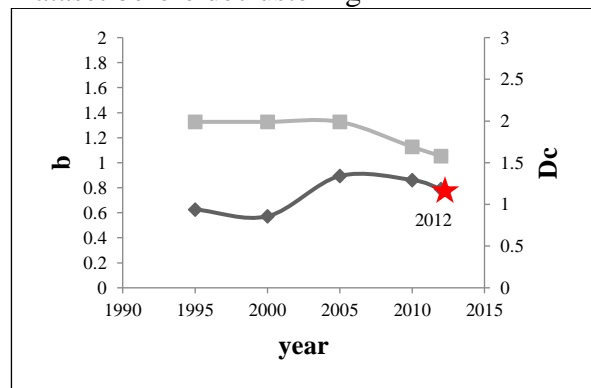


(e) Event 5

Dataset after declustering

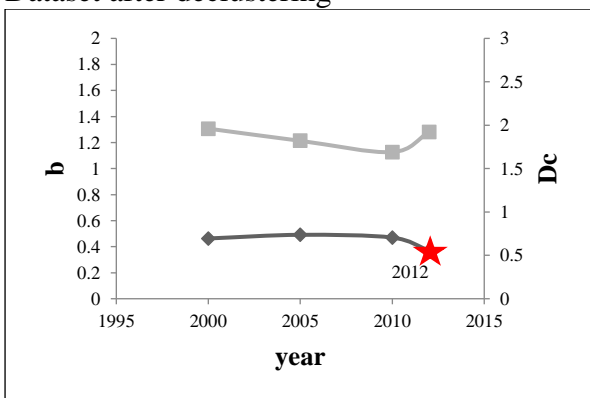


Dataset before declustering

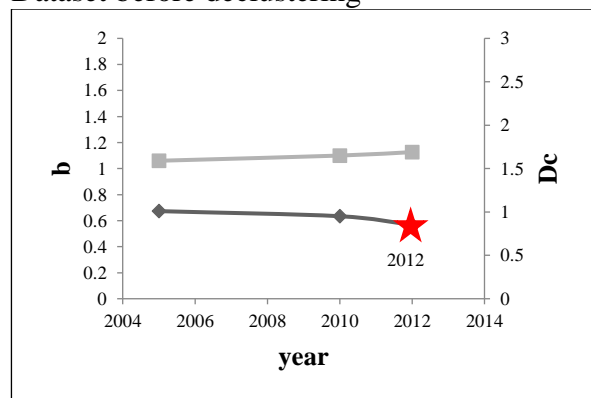


(f) Event 6

Dataset after declustering



Dataset before declustering



b ——— Dc ———

Figure 4. Temporal variation of b and Dc value evaluated from both after- and before-declustering dataset recorded along the SASZ. Stars denotes the time of recognized earthquake occurrence.

5. b value and Dc value relationship

The b-Dc relationship has been suggested as an effective indicator of seismic hazards (Bayrak and Bayrak, 2011; 2012). The b-Dc can either be a

positive or negative correlation. For example, Bayrak and Bayrak (2012) studied about Western Anatolia and the relationship of b and Dc was negative correlation. Bhattacharya et al. (2010)

studied the earthquake source zone in Northeast India and the correlation of b and D_c was positive. This research, there

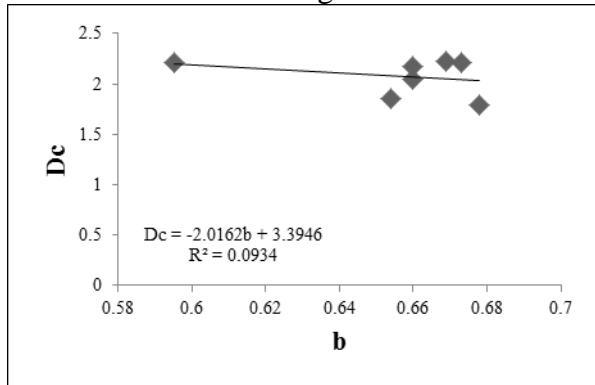
are 6 earthquakes to study (Figures 5a-f). The b - D_c relationships of 6 earthquakes are all negative (Table 2).

Table 2. Sumarize of b - D_c relationship of 6 earthquake case study.

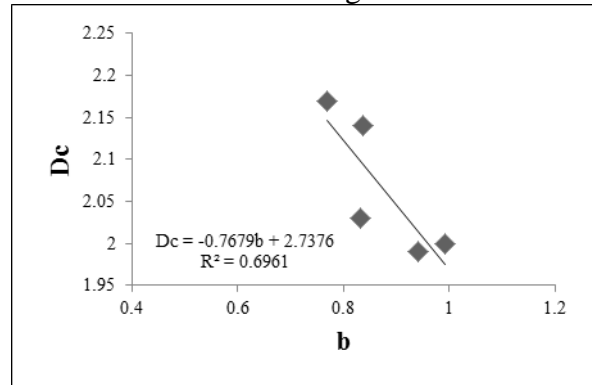
event	After declustering dataset			Before declustering dataset		
	b- D_c	r^2	relation	b- D_c	r^2	relation
1	$D_c = -2.0162b + 3.3946$	0.0934	negative	$D_c = -0.7679b + 2.7376$	0.6961	negative
2	$D_c = -1.2591b + 2.9706$	0.9651	negative	$D_c = -0.1543b + 2.134$	0.0294	negative
3	$D_c = -4.4842b + 5.1347$	0.2157	negative	$D_c = -0.3842b + 2.2769$	0.2052	negative
4	$D_c = -1.5274b + 3.8498$	0.3781	negative	$D_c = -0.1255b + 2.1932$	0.101	negative
5	$D_c = -0.4259b + 2.3426$	0.4681	negative	$D_c = -0.6208b + 2.313$	0.2001	negative
6	$D_c = -0.9575b + 2.2757$	0.25	negative	$D_c = -2.155b + 3.0385$	0.8942	negative

(a) Event 1

Dataset after declustering

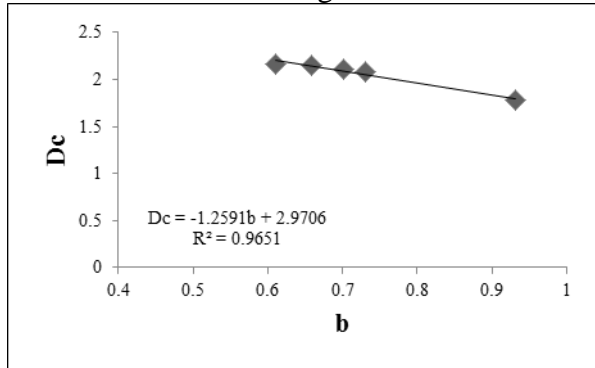


Dataset before declustering

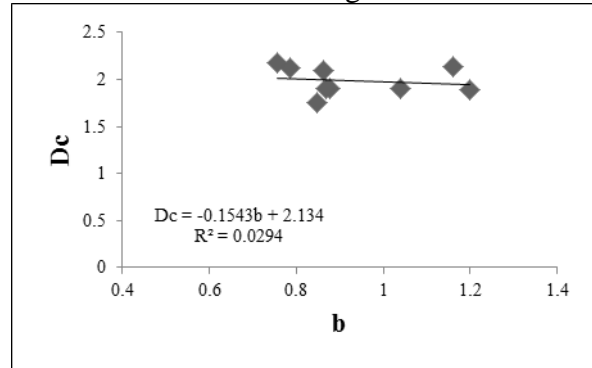


(b) Event 2

Dataset after declustering



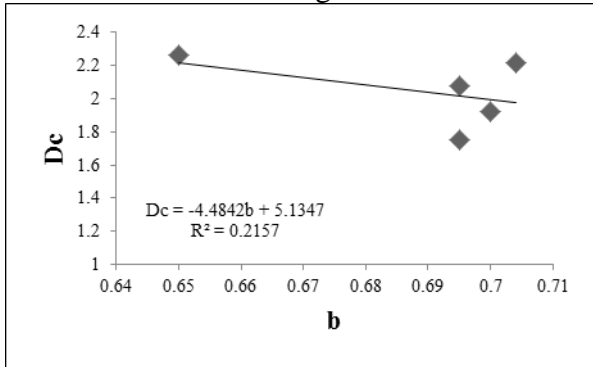
Dataset before declustering



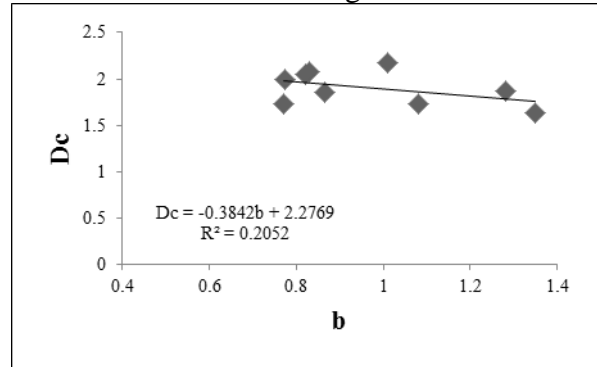


(c) Event 3

Dataset after declustering

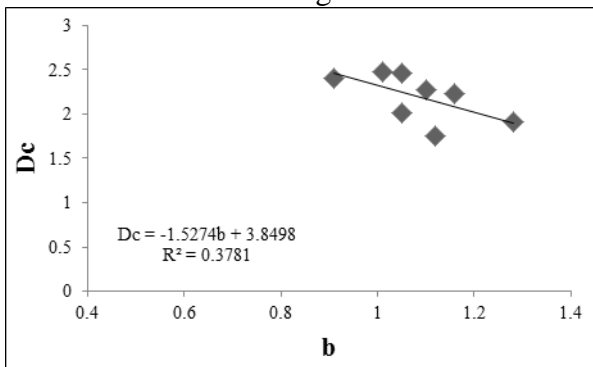


Dataset before declustering

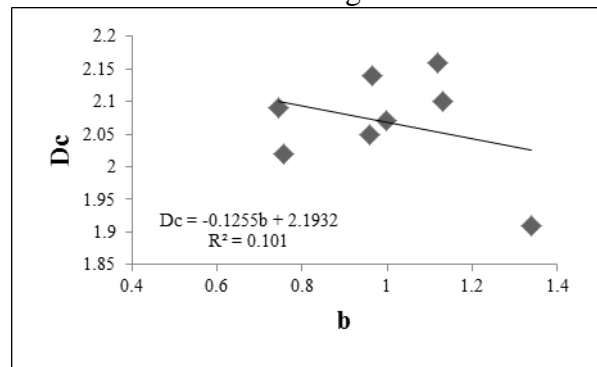


(d) Event 4

Dataset after declustering

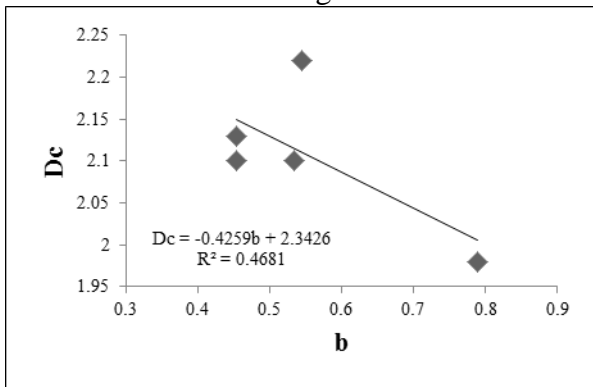


Dataset before declustering

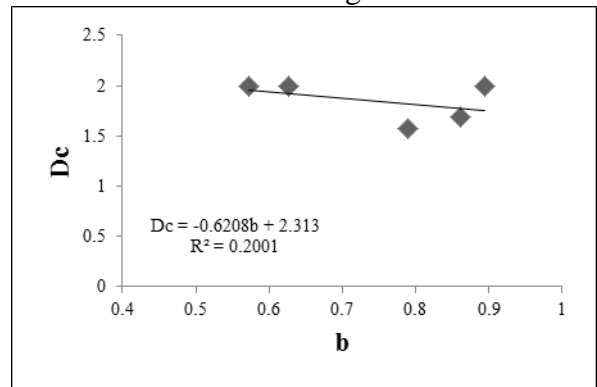


(e) Event 5

Dataset after declustering

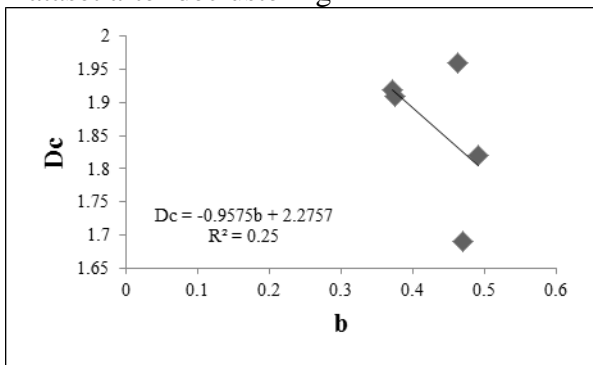


Dataset before declustering



(f) Event 6

Dataset after declustering



Dataset before declustering

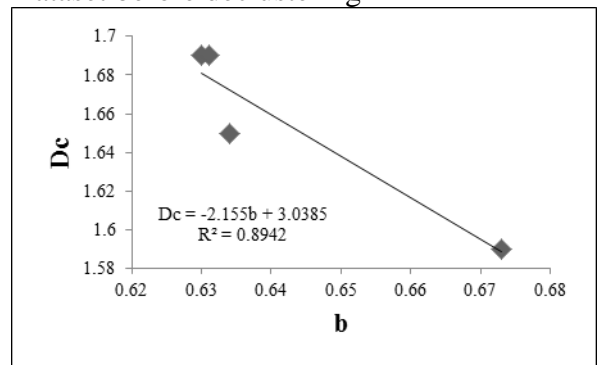


Figure 5. Graphs showing relationship between b and D_c values of 6 earthquakes analyzed from i) after-declustering dataset and ii) before declustering dataset.

6. Conclusion

This research focuses on the temporal investigations of the seismicity posed along the SASZ. In particular the seismicity data within 300-km radius extended from each earthquake with $M_w \geq 8.0$. In order to clarify the earthquake activities and patterns, both b and D_c values were investigated. In addition, both b and D_c values were analyzed based on 2 dataset, i.e., before- and after-declustering dataset

As a result, when both after- and before-declustering datasets were compared, we found that b value of after-declustering dataset are in the range of 1.28-0.36 before all of 6 earthquake case study occurred. It means that there is a cumulative high stress on the earthquake areas. For the b value of before declustering was low at the range of 1.35-0.56. However, the b value of before declustering was higher than after-declustering dataset.

In the part of analyzing D_c value, it was found that both before and after-declustering dataset had conform fairly. It clearly showed that the areas of 6 earthquakes had D_c value nearly reached 2. It means that the characteristic of earthquake is “plane” sources. The b value and D_c value relationship of before and after-declustering dataset had correlation with 6 earthquakes. It reveals b value and D_c value have the negative relationship. So, It showed that the result of analyzing b - D_c relationship related with result of the overall area (SASZ).

Therefore, both before- and after-declustering datasets can use to study the b and D_c values because the comparison of graph between before and after-declustering dataset showed that the result was very similar.

7. Acknowledgments

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